DOCUMENT RESUME

ED 480 462 SE 068 318

TITLE	Algebraic Skills and Strategies for Elementary Teachers and Students.
INSTITUTION	Wisconsin Univ., Madison. National Center for Improving Student Learning and Achievement in Mathematics and Science.
SPONS AGENCY	Office of Educational Research and Improvement (ED), Washington, DC.
PUB DATE	2003-00-00
NOTE	12p.
CONTRACT	R305A960007
AVAILABLE FROM	National Center for Improving Student Learning and Achievement in Mathematics and Science, Wisconsin Center for Education Research, University of Wisconsin-Madison, School of Education, 1025 W. Johnson Street, Madison, WI 53706. Tel: 608-263-7582; e-mail: ncisla@education.wisc.edu; Web site: http://www.wcer.wisc.edu/ ncisla.
PUB TYPE	Collected Works - Serials (022) -- Guides - Non-Classroom (055)
JOURNAL CIT	In Brief; v3 n1 Sum 2003
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	*Algebra; Arithmetic; Cognitive Processes; Elementary Education; *Elementary School Mathematics; Elementary School Students; Elementary School Teachers; Mathematics Instruction; Professional Development; Teacher Improvement; Thinking Skills

ABSTRACT

Research shows that introducing basic forms of algebraic reasoning in elementary school enhances children's learning of arithmetic. In this guide, a research and a professional development project are described that have taken a practical approach to introducing algebraic reasoning to elementary students in Massachusetts. (Author/SOE)

Reproductions supplied by EDRS are the best that can be made
from the original document.

ED 480 462

[Algebraic Skills and Strategies for Elementary Teachers and Students]

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☐ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

BEST COPY AVAILABLE

SE 063 310

NATIONAL CENTER FOR IMPROVING STUDENT LEARNING & ACHIEVEMENT IN MATHEMATICS & SCIENCE

in Brief

K-12 Mathematics & Science
RESEARCH & IMPLICATIONS
FOR POLICYMAKERS, EDUCATORS & RESEARCHERS
SEEKING TO IMPROVE STUDENT LEARNING & ACHIEVEMENT

Algebraic Skills & Strategies for Elementary Teachers & Students

In the United States, students typically are not introduced to algebra until the 8th-10th grades, but research shows that introducing basic forms of algebraic reasoning in elementary school enhances children's learning of arithmetic. The National Center for Improving Student Learning and Achievement in Mathematics and Science (NCISLA) has found that *young children* can learn to reason algebraically.[1] Described here is a research and professional development project that has taken a practical approach to introducing algebraic reasoning to elementary students in Massachusetts.

As part of a Massachusetts district-wide school improvement plan, a NCISLA research team has been leading a professional development program focused on strategies to integrate algebraic reasoning into teachers' current lessons as early as the first grade. Instead of treating algebra as an add-on to the curriculum, the researchers recommend "algebrafying" the curriculum, an approach that takes into account teachers' challenging work environment as well as traditional elementary teacher preparation. A case study of students' learning, including data on students' performance on the Massachusetts Comprehensive Assessment System (MCAS), provides preliminary evidence that this strategy improves young students' learning and achievement in mathematics.

"Algebrafying" Elementary Mathematics Instruction

The goal of this Massachusetts project has been to treat elementary school mathematics, especially arithmetic, in a more algebraic way. Jim Kaput and Maria Blanton, NCISLA researchers who lead this project, offer the following as an example of a traditional arithmetic problem that can be transformed into one that develops students' algebraic reasoning and provides students with skills to solve new and more challenging problems:

If 5 people in a group shake hands with each other once, how many handshakes will there be? What if there are 6 people in the group? 7 people? 8 people? 20 people? Write a number sentence that shows the total number of handshakes. How did you get your solution? Show your solution on paper. (See box on page 4, Focus on the "Handshake Problem".)[2]

"Jan",[3] the case study teacher, used this problem as a basis for a lesson for her third-grade students. After dividing her students into groups, Jan asked them to act out the problem and determine the total number of handshakes in each group. She directed students to consider helpful ways to keep track of their data (the number of handshakes) and to model their process arithmetically. Students formed number sentences such as 7 + 6 + 5 + 4 + 3 + 2 + 1, found ways to group numbers (e. g., grouping the number of handshakes occurring when there were 5 people in a group and then counting on to determine the number for a group of 8 people), and identified patterns that ultimately led them to use multiplication to solve the problem. By writing number sentences and then working with the form of the number sentences (rather than simply computing a number for each case), the students came to work with numbers and operations in an algebraic way.[4]

[1] *See also* in Brief *(2000) article on the NCISLA early algebra research project headed by NCISLA director and researcher Tom Carpenter.*
[2] *For an earlier version of the handshake problem that appears frequently in professional literature, see Yazema, Adams, & Cagle (2000).*
[3] *"Jan" is a pseudonym.*
[4] *A video of this classroom episode is available through Annenberg/CPB and also on the forthcoming NCISLA CD-ROM, Powerful Practices in Mathematics & Science: Modeling, Generalizing, & Justifying, to be distributed by the North Central Eisenhower Mathematics and Science Consortium at the North Central Regional Educational Laboratory. (See box on page 4.)*

LB2361.5 .I53 no.8 2000

Examining Teacher Practice & Student Learning

Kaput and Blanton's team conducted a year-long case study of Jan's class to document student learning of algebraic reasoning. The researchers also documented the effects of this professional development program on Jan's instructional practice as well as on student achievement on selected items from the fourth-grade MCAS (in comparison to a control group and the district).

Teacher practice integrating algebraic reasoning. Blanton observed Jan's 90-minute third-grade math class about 3 days a week during the course of an academic year (38 visits total). Following aspects of the design experiment approach,[5] Blanton worked closely with Jan, occasionally co-teaching the course. Through collecting Jan's reflections and examples of student work and visiting Jan's classroom, the researchers documented the ways in which Jan integrated algebraic reasoning into her instruction (Blanton & Kaput, 2002).

The researchers were interested in the diversity and frequency of Jan's integration of algebraic reasoning. They identified a total of 206 instances of algebraic reasoning covering 12 different types of algebraic practice. Of these episodes, 132 (65 percent) were characterized as instances in which Jan spontaneously crafted instruction that required students to reason algebraically. Blanton and Kaput considered this significant, indicating that Jan's knowledge of algebraic reasoning enabled her to see ways that algebra could be integrated into arithmetic lessons. The remaining 74 episodes included planned activities, most of which (63 instances) were activities Jan developed from her own instructional resources.

Student learning. At the conclusion of the study, the researchers administered a set of 14 test items from the fourth-grade MCAS to the 14 third-grade students present in Jan's class the day of the test—and compared the results to a control group of third-grade students in the same school.[6] The results offer some preliminary evidence to support the value of the algebrafication and professional development strategy implemented in Jan's classroom (Blanton & Kaput, in press-a).

Jan's experimental group performed better than the control group on 11 of the 14 selected test items (4 of which were significant at alpha = 0.05). Jan's students scored higher than the control group on 6 out of the 7 items that the researchers identified as being deeply algebraic in nature. These problems, such as the one in Figure 1, required students to find patterns, understand whole-number properties, and identify unknown quantities in a number sentence.

In addition, the results from Jan's *third*-grade classroom also were compared to the performance of the district's *fourth*-grade students on the MCAS: A higher percentage of students in Jan's class scored at the "advanced" and "proficient" levels (see Blanton & Kaput, in press-a). These results are noteworthy given that many of Jan's students were from homes where English was either not the primary language or not spoken at all. The socioeconomic status of students in Jan's third-grade class was also lower than average for the district.[7]

Enhancing Student Learning Through Algebraic Tasks

Algebraic tasks like the handshake problem can help students learn to—

- Represent data.
- Construct a number sentence that models a phenomenon.
- Examine how variations in a phenomenon affect the number sentence.
- Use a number sentence to reason algebraically about a problem.
- Understand the properties of whole numbers and the number system.
- Understand the relationships between operations in order to facilitate computation (e.g., recognize repeated addition as multiplication).

How many of the smallest squares will be in Figure 5 if this pattern continues?

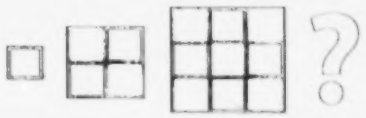

FIGURE 1. Sample MCAS problem identified as "deeply algebraic".

Professional Development for Instructional Change

The professional development project provides teachers and administrators a practical approach to changing elementary mathematics instruction in ways that build students' algebraic thinking. Now in their seventh year of the project, Kaput and Blanton have worked with administrators in an academically underachieving Massachusetts school district[8] to implement a research and professional development program as part of the improvement plan required by the Massachusetts Department of Education. The program was touted as "exemplary" by writers of the No Child Left Behind Act Summary of the 2001 Reauthorization Conference Report of the U.S. Senate Health, Education, Labor, and Pensions Committee. The program addresses both teachers' and administrators' needs through a leadership academy and professional development seminars.

The leadership academy. Kaput and Blanton consider administrative support, especially at the school level, crucial to the success of teachers' professional development. In order to build the capacities of principals to support instructional change, the researchers and district superintendent held seminars for all K-5 principals and curriculum coordinators once a month for one semester. Given the principals' personnel responsibilities, the leadership academy was designed to help administrators understand the algebrafication strategy and how that approach could inform hiring decisions, teacher evaluation, and day-to-day supervision. The sessions addressed practical concerns such as the implementation of new state curriculum frameworks and ways to allocate more time for teachers to

5 *For more on design experiments, see Cobb, Confrey, diSessa, Lehrer, & Schauble (2003).*
6 *The teacher in the control classroom did not participate in the professional development program.*
7 *About 75% of the students in Jan's class were on free lunch, and 15% on reduced lunch, 65% were from families for whom English was a second language; 25% had no parent living at home.*
8 *The district includes 30 relatively small elementary schools.*

BEST COPY AVAILABLE

RESOURCES *for* TEACHERS & POLICYMAKERS

Developing Children's Algebraic Reasoning

Algebraic thinking focus issue [Special issue]. (1997). *Teaching Children Mathematics, 3* (6).

Blanton, M., & Kaput, J. (in press). Developing elementary teachers' algebra "eyes and ears". *Teaching Children Mathematics.*

Cai, J. (1998). Developing algebraic reasoning in the elementary grades. *Teaching Children Mathematics, 5* (4), 225-229.

Carpenter, T. C., Franke, M. L., & Levi, L. (2003). *Thinking mathematically.* Portsmouth, NH: Heinemann.

Schifter, D. (1999). Reasoning about operations: Early algebraic thinking, Grades K-6. In L. Stiff & F. Curio, (Eds.), *Mathematical reasoning, K-12: 1999 NCTM yearbook* (pp. 62-81). Reston, VA: National Council of Teachers of Mathematics.

Ways that Schools Can Support Change

Gamoran, A., Anderson, C. W., Quiroz, P. A., Secada, W. G., Williams, T., Ashmann, S. (2003). ***Transforming teaching in mathematics and science: How schools and districts can support change.*** New York: Teachers College Press.

Supporting professional development and teaching for understanding. (2002, Fall). ***in Brief.*** (Available at http://***www.wcer.wisc.edu/ncisla/publications***)

NATIONAL CENTER FOR IMPROVING STUDENT LEARNING & ACHIEVEMENT IN MATHEMATICS & SCIENCE

in Brief

K-12 Mathematics & Science POLICY CONSIDERATIONS

Algebraic Strategies for Elementary Grades

In light of concerns about student performance on state, national, and international assessments, educators are reexamining approaches to mathematics curricula and teaching practices. The approach described in this issue of *in Brief* suggests ways that teachers can engage students in algebraic reasoning as they learn arithmetic. Importantly, the approach takes into account the capacities and constraints of teachers, administrators, and available instructional materials. Described below are several ways that administrators can support teachers in this approach.

Supporting teacher learning and professional development. Administrative support of content-rich professional development that builds teachers' knowledge and their skills in developing algebraic problems and activities is a key part of transforming elementary mathematics teaching. Building teachers' "algebra eyes and ears" requires an initial focus on content knowledge and skills that often means engaging expertise available in the district, state, or community. When teachers learn ways to transform arithmetic problems into algebraic reasoning problems, they also learn how to identify and organize opportunities for integrating algebraic reasoning in their classrooms, while simultaneously developing their own instructional resource base.

Building a professional community network. In the Massachusetts district, NCISLA researchers Blanton and Kaput are finding that a teacher community is critical in supporting teachers as they transform their practice. Developing a community takes time (often 3 to 4 years), and such a community and its growth are sensitive to changes in leadership and teacher attrition. Establishing a network, however, can prevent isolation and support teachers as they work to change their practice over time. Blanton and Kaput propose establishing what they describe as a professional community network (Blanton & Kaput, in press-c), which would include several interconnected but distinct communities that have parallel purposes focused on a common goal. In the district described here, three communities are evolving: (a) teacher communities, (b) principal communities, and (c) communities based on teacher-principal partnerships. The researchers found that building connections among these communities allowed change to occur in a network of mutually supportive relationships.

Linking leadership to professional development goals. The leadership academy, conducted as part of the Massachusetts professional development program, addressed areas in which leaders could support teachers in integrating algebra into their practice. Activities were patterned after those used in the teacher seminars in order to provide participants with experience in algebraic reasoning. (Teachers shared their classroom experiences with principals and district leaders.) Specifically, the sessions focused on—

- Understanding and supporting algebraic reasoning and practice so that the evaluation and hiring of teachers reflected the needed move in this direction.
- Assisting administrators in restructuring the school day to allow for ongoing teacher collaboration.
- Enabling teachers to promote mathematics literacy on a school-wide basis and to integrate it into other district initiatives.

Common among the participating princpals was their commitment to—

- Work with teachers to enhance their professional development.
- Preserve teachers' autonomous role in leading and designing professional development.
- Share decision-making authority with teachers on issues that affected the school community.

Developing congruency across educational initiatives. Blanton and Kaput define *developing professional congruency* as identifying and strengthening ways in which teachers' professional obligations and interests can support each other (Blanton & Kaput, in press-c). Because the algebrafication strategy is about transforming practice, not adhering to a particular curriculum, the approach can be applied in various contexts. For example, Blanton and Kaput worked to integrate MCAS items into the project as a way to increase professional congruency. The pressures across the district to perform well on this state assessment made coverage of this material relevant to the teachers. Thus, one of the tasks for teachers in the program was to algebrafy MCAS by identifying test items that involved (or could be extended to involve) algebraic reasoning and then include those items in their daily instruction.

Similarly, the Massachusetts district was able to connect the algebrafication approach with the district's literacy initiative and, more recently, with another mathematics professional development program adopted by the district. Rather than compete with the literacy initiative, which was long-running and had substantial funding, the team sought ways to find synergies between the two programs—each of which emphasized active meaning-making and purposeful expression of ideas. Several teacher-leaders collaborated in developing a year-long professional development agenda that would explore algebraic reasoning in the context of the literacy initiative. As part of this collaboration, they looked for children's literature that could be used in conjunction with existing algebraic tasks or in the creation of new tasks. They sought to connect initiatives in order to provide students additional ways to access algebraic tasks and to increase the potential frequency with which teachers could integrate algebraic reasoning into their classrooms.

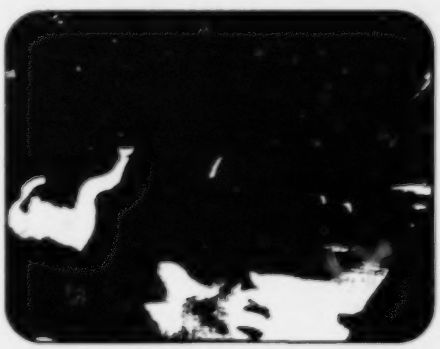

PHOTO CREDIT: Annenberg/CPB

NCISLA • Wisconsin Center for Education Research • School of Education, University of Wisconsin-Madison 1025 West Johnson Street • Madison, Wisconsin 53706
Phone (608) 263-7582 E-mail: ncisla@education.wisc.edu Web site: http://www.wcer.wisc.edu/ncisla

in Brief

K-12 Mathematics & Science POLICY CONSIDERATIONS

Algebraic Strategies for Elementary Grades

The research described in this issue of *in Brief* shows young students' potential for reasoning algebraically in conjunction with their learning of arithmetic. The professional development approach taken by the research team is described here in more detail to provide educators a deeper look at the ways these teachers worked to change their instructional practice.

Establishing professional development seminars. The cross-grade seminars provided elementary teachers both time and a place to reflect on a common base of problems, which they customized and solved individually. As they progressed in their learning of the ways algebraic reasoning related to the teaching and learning of arithmetic, the teachers tailored the problems to their own grade levels, tried them with their students, and then shared the results with their colleagues. Through the seminars, teachers gained insight into the ways problems—and students' problem solving and reasoning strategies—differed across grade levels.

Designing challenging problems. In the cross-grade seminars, teachers designed challenging mathematical problems that provided students with experiences in generalizing and formalizing patterns and relationships, as well as in justifying conjectures. The problems were selected based on the extent to which they—

- Addressed important mathematical ideas.
- Were approachable at different levels and with different representations.
- Had potential to generate rich conversations.
- Involved substantial quantitative reasoning and computation.

Through collaboratively solving such problems and redesigning tasks for students, the teachers and researchers outlined several principles for designing algebraic reasoning (Blanton & Kaput, in press-b).

These principles guided teachers in developing classroom tasks that—

- Involved **sequences of computations** yielding numerical patterns that served to engage students arithmetically.
- Promoted the use of **non-executed number sentences** (e.g., 0 + 1 + 2 + 3 + 4 + ... + 18 + 19) as objects for reasoning algebraically, rather than simply occasions to compute.
- Facilitated the **algebraic use of number**. For example, a teacher might use so large a number that arithmetic becomes an inefficient way to solve the problem (e.g., 20 in the third-graders' handshake problem), and students are encouraged to think about patterns and relationships (see page 1).
- Could be **sequenced by a set of events or objects**, particularly through figural or physical enactment, such as shaking hands. This provided a concrete starting point for building patterns and relationships among mathematical concepts.

Transforming instructional materials. The seminars often began with the question, "How can this one-numerical-answer problem be transformed into an algebraic-reasoning problem that involves building and expressing a pattern or generalization?" This transformation was typically accomplished by varying one of the numerical "givens" of the problem and then examining the patterns that emerged through developing a series of number sentences and calculations.

For example, in the word problem –

I want to buy a tee shirt that costs $14 and have $8 saved already. How much more money do I need to earn to buy the shirt?

A teacher could vary the "givens" of a word problem to make it an algebraic task perhaps by varying the cost of the item:

Suppose it cost $15, $16, $17, or $26. Using P for the price of the item I want to buy, write a number sentence that describes how much more money I need in order to buy the item.

Or, the teacher could vary the conditions of the problem:

Assuming I make $2 per hour for babysitting, how many hours do I need to work to have enough money to buy the shirt that costs $14? $20? $P? What if I earned $3 per hour? How many hours do I need to work to buy the $14 shirt?

(Visit the NCISLA website at http://www.wcer.wisc.edu/ncisla, Teacher Resources, for sample Algebrafied Arithmetic Tasks.)

Developing classroom norms. A key component involved in students learning to reason algebraically is active student involvement in proposing mathematical conjectures and justifying their reasoning. Throughout the seminars, teachers were supported in finding ways to create a classroom culture that fostered the kind of discussions needed to develop students' algebraic reasoning skills.

Detailed analysis of the case study data (Blanton & Kaput, 2002) revealed several characteristics of teaching practice that integrated algebraic reasoning and mathematics. In her classroom, "Jan's" increased sensitivity to algebraic reasoning meant she was able to thread algebraic themes into her conversations with students over sustained periods of time. Specifically, she was able to—

- Engage in a spontaneous and planned algebraic treatment of number.
- Integrate algebraic processes into a single task.
- Generalize an activity to introduce algebraic themes.

Jan engaged students in the algebraic treatment of number through—

- Discussion of number properties such as odd-even parity or commutativity.
- Use of symbols to represent unknown quantities.
- Variation of tasks along one dimension to generate numerical patterns.

NCISLA — Wisconsin Center for Education Research — School of Education, University of Wisconsin-Madison — 1025 West Johnson Street — Madison, Wisconsin 53706

collaborate and participate in professional development activities. The seminars also provided principals and coordinators an opportunity to discuss the approach with teacher-leaders. In their first session, participants solved the handshake problem before viewing a video of the third-grade students solving the same problem. The principals were impressed by their students' apparently high capacity for mathematical thinking, given the students' prior performance on state assessments.

Professional development. Because this approach to algebrafying mathematics instruction is outside most elementary teachers' experience, biweekly after-school seminars focused on developing teachers' algebra "eyes and ears." During each academic year, approximately 50 teachers from 16 schools across Grades K–5 participated in seminars that were led by teams of peer teacher-leaders (usually two) who had undergone at least one year of training with Kaput and Blanton in similarly structured seminars. The researchers met monthly with these teacher-leaders to collect data and assess the effects of the seminar activities on teacher practice.

Supporting Instructional Change

The researchers identified the following factors that support the growth of a teacher community:

- Establishing grade-level school-based study groups led by teacher-leaders.
- Engaging teacher-participants for a minimum of 1 year (2 or more years in most cases).
- Conducting seminars that emphasize solving mathematical problems and understanding students' thinking, then using these as a catalyst for thinking about teaching practice.
- Using the resources teachers generate as the basis for a shared, growing set of materials.
- Aligning the implementation of the professional development program with other initiatives.
- Integrating statewide assessment tasks into the teacher resource base.

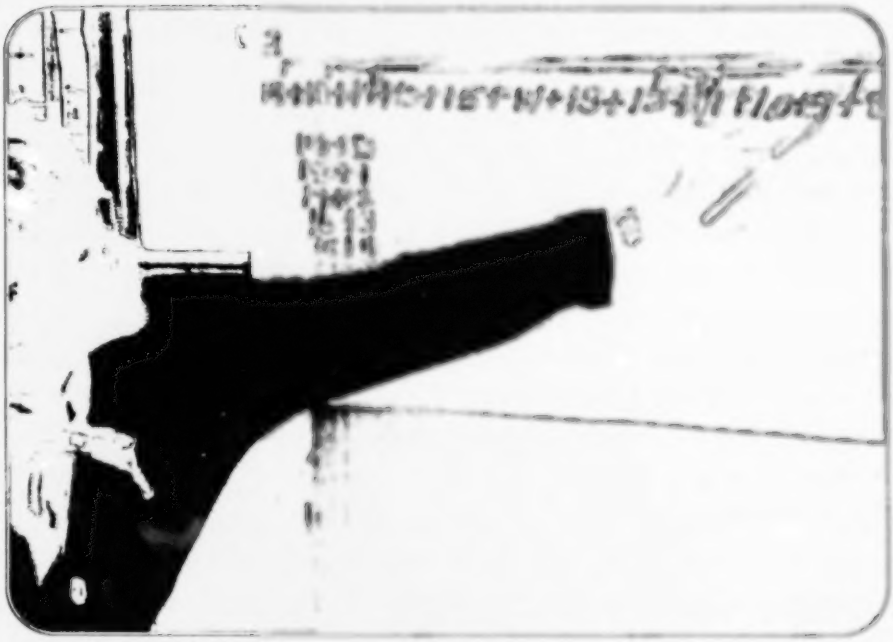

PHOTO CREDIT: Annenberg/CPB

When Jan's Grade 3 students generated a number sentence for a group of 20 people shaking hands, the students organized the numbers into 10 pairs, with the sum of each being 19. (See Focus on the "Handshake Problem," page 4).

Teacher Learning Through Professional Development

Kaput and Blanton's approach to professional development takes into account constraints on teachers—namely a prior orientation that focuses on arithmetic and computation without integration of algebraic reasoning, as well as textbooks and instructional materials that take a similar approach. At the same time, the researchers recognize that teachers' capacity for mathematical and pedagogical growth can offset the challenges presented by those constraints.

The professional development seminars, which purposely involved teachers from across multiple grade levels, were structured around the three activities discussed here. (More detail about these activities is provided in the Teacher Considerations insert.)

Customizing and solving problems. The goal of this program was to make aspects of algebraic reasoning (such as generalizing and formalizing) part of mainstream instruction rather than a form of occasional enrichment. Elementary teachers typically have little experience with generalizing and formalizing—activities considered to be the heart of algebraic reasoning. For this reason, the professional development approach engaged teachers in rich mathematical experiences embodying these activities.

Rather than implementing a new instructional program and curriculum, this research and professional development program focused on enabling teachers to enhance their existing mathematics resource base and incorporate instructional practices to promote students' algebraic reasoning. Specifically, teachers were introduced to ways in which selected arithmetic problems could be transformed into algebraic-reasoning problems. By using teachers' at-hand instructional materials as a base for this activity, algebrafying became part of teachers' daily practice. The process of finding and modifying other problems helped build the skills needed to continue long-term growth and development beyond the seminars.

Group discussion and teacher observations of their students' work allowed for comparison across grade levels, giving teachers an opportunity to learn more about students' growth trajectories and develop strategies for adjusting activities accordingly. In later iterations of the

seminar, locally produced teacher tested activities from one year became a resource for new teacher participants in following years. The researchers considered this important as a way to help teachers see these materials as their own rather than as a set of externally provided materials to be "implemented."

Providing readings to support reflections. The research team provided readings that offered teachers an opportunity to reflect on the ways their approaches to mathematics instruction differed from, or were similar to, other instructional practices detailed in the education research literature. (See the Resources for Teachers and Policymakers listed on the bookmark in this publication.)

Addressing classroom norms. In the seminars, teachers considered ways to foster classroom discussions that would develop students' algebraic reasoning skills. The processes of building patterns, making conjectures, and generalizing and justifying mathematical facts and relationships are at the core of the algebraic reasoning strategy. Teachers focused on these issues and considered questions such as whether a culture of inquiry was developing in their classrooms, what the classroom norms for argumentation were, and whether students questioned each other and expected justification of mathematical statements. Teachers also discussed the extent to which their students were learning computation and justification skills, the differences in instructional approaches and experiences, and the difficulties involved in changing practice.

Next Steps

The separation of algebra from early mathematics instruction dates back to the 1400s (Swetz, 1987), but this separation is not appropriate for contemporary mathematics instruction. Researchers suggest that, when introduced in elementary grades, algebraic reasoning paves the way for more advanced learning in middle and high school and strengthens student understanding of arithmetic. This issue of *in Brief* offers a glimpse into a content-rich professional development program that develops teachers' "algebra eyes and ears." When considering long-term improvement plans, schools and districts should consider allocating professional development resources toward similar approaches, which do not necessarily require the purchase of new curricula or instructional programs (see insert on Policy Considerations).

Focus on—the "Handshake Problem"

The handshake problem engaged "Jan's" students in algebraic reasoning. Initially, the third-grade students acted out the multiple handshakes for smaller-sized groups and carefully recorded their data for different numbers of people.

When she gathered the students into a larger group, Jan led the students to generate a number sentence for a group of 20 people. The students came up with "19 + 18 + 17 + 16 + ... + 2 + 1 + 0".

At that point, Jan asked the students to consider a way to "change the order of these numbers and make them easier to add up." The students realized that they could organize the numbers into 10 pairs, with the sum of each being 19 (e.g., 19 + 0, 18 + 1, 17 + 2). The following discussion ensued:

JAN: Now I've got to add up all these 19s. What is this?

STUDENT: Repeated addition. You could do times.

JAN: I could do times?

STUDENT: 19 times 10 [pairs]—190.

JAN: How did you figure that out so quickly?

STUDENT: I just changed that to 9 and added a zero.

JAN: Why?

The student explained that he had made the "1 into a 100 and 9 into 90" by using 10 times 10 to get 100, and 9 times 10 to get 90, then added.

Through the handshake problem, Jan's students came to recognize a pattern that led to an important generalization about how to calculate the sum of any arithmetic series. In solving a particular problem, they generated and justified a procedure that can be applied to find the sum of any arithmetic series.

To View a Video of Students Solving the "Handshake Problem"

Visit Annenberg/CPB at http://www.learner.org/catalog, or call **1-800-LEARNER** and ask about the ***"Looking at Learning... Again, Part 2"*** video series.

This video is also featured in a forthcoming NCISLA multimedia package — ***Powerful Practices in Mathematics and Science: Modeling, Generalizing, and Justifying*** — to be released through the North Central Eisenhower Mathematics and Science Consortium at the North Central Educational Laboratory.

To order Powerful Practices, e-mail ***barbara.youngren@ncrel.org.***

Building Algebraic Reasoning Q&A

What's Different About an "Algebrafied" Approach to Arithmetic?

Traditional elementary school mathematics curricula focus on isolated computations and the solution of single, self-contained problems—usually with a single number answer. An "algebrafied" approach, on the other hand, focuses on solving meaningfully related families of problems, for which the "answers" typically are student-generated generalizations. When students make generalizations, they account for the characteristics common across problems, the solution methods that apply to such problems, and how far the solution methods extend. Along the way, students regularly practice computational skills through solving arithmetic problems. Importantly, instead of primarily solving numerous arithmetic "practice exercises" (such as those found on typical worksheets), arithmetic problems are embedded in tasks meaningful to the students whose goal is to build, justify, and express generalizations (Kaput, 1999). Through this strategy, students learn to represent generalizations and justifications with symbols, graphs, charts, or diagrams. This approach allows them to solve increasingly complex problems and lays the foundation for more complex mathematics, including algebra, in later grades. For teachers, this approach means identifying ways to modify and develop problems while searching for and being explicit about features shared across problems.

An "algebrafied" approach focuses on solving meaningfully related families of problems, for which the "answers" typically are student-generated generalizations.

What Are Important Elements of a Curriculum That Supports Algebraic Reasoning?

GENERALIZING. The process of developing and proposing general mathematical statements concerning the structure, properties, or relationships that underlie mathematical ideas.

FORMALIZING. The process of representing mathematical generalizations, with such formalizations ranging from the use of everyday language to formal, symbolic rules. Students progress in their mathematical abilities as they express their ideas in increasingly formal, mathematical ways. For example, a student might first express a pattern in everyday language such as "add two every time" and then go on to represent that pattern in a more symbolic form such as "$+ 2$".

JUSTIFYING CONJECTURES. The process of developing mathematical arguments to explore and critique the validity of mathematical claims. An important first step in this process is that students understand that mathematical claims can and should be justified.

PHOTO CREDIT: Annenberg/CPB

Looking for sample tasks to build students' algebraic reasoning?

Check out the elementary school resources at NCISLA's Teacher Resources Page:

www.wcer.wisc.edu/ncisla/teachers/index.html.

The PDF document, "Algebrafied" Arithmetic Tasks, features tasks that have been reframed to build elementary students' algebraic thinking.

For More Information

This issue of *in Brief* is based on research by Maria Blanton and Jim Kaput that is reported in two forthcoming books, *Understanding Mathematics and Science Matters*, edited by T. Romberg, T. Carpenter, and F. Dremock, and *Everyday Matters in Science and Mathematics*, edited by R. Nemirovsky, B. Warren, A. Rosebery, and J. Solomon. For additional publications by Maria Blanton and Jim Kaput, see ***http://www.wcer.wisc.edu/ncisla/publications*** and ***http://www.wcer.wisc.edu/ncisla/teachers***.

ABOUT *in Brief*

This publication and the research reported herein are supported under the Educational Research and Development Centers Program, PR/Award Number R305A960007, as administered by the Office of Educational Research and Improvement, U.S. Department of Education, and the Wisconsin Center for Education Research at the University of Wisconsin–Madison. The opinions expressed herein are those of the featured researchers and authors and do not necessarily reflect the views of the supporting agencies.

ABOUT *NCISLA*

NCISLA is a university-based research center focusing on K–12 mathematics and science education. Building on several years of research, Center researchers collaborate with schools and teachers to create and study instructional approaches that support and improve student learning and understanding of mathematics and science. Through research and development, the Center is identifying new professional development models and ways that schools can better support teacher professional development and student learning.

NCISLA Director: Thomas P. Carpenter

NCISLA Associate Directors: Jim Stewart & Paul Cobb

NCISLA Communication Director: Susan Smetzer Anderson

Writer: Valerie Farnsworth

Editor: Fae Dremock

Designer: Linda Endlich, IMDC Graphics

Permission to copy is not necessary.
This publication is free on request.

REFERENCES

Blanton, M., & Kaput, J. L. (in press-a). Algebrafying the elementary mathematics experience in a teacher-centered, systemic way. In T. Romberg, T. Carpenter, & F. Dremock (Eds.), *Understanding mathematics and science matters*. Mahwah, NJ: Erlbaum.

Blanton, M., & Kaput, J. L. (in press-b). Design principles for instructional contexts that support students' transition from arithmetic to algebraic reasoning: Elements of task and culture. In R. Nemirovsky, B. Warren, A. Rosebery, & J. Solomon (Eds.), *Everyday matters in science and mathematics*. Mahwah, NJ: Erlbaum.

Blanton, M., & Kaput, J. L. (in press-c) Developing elementary teachers' algebra "eyes and ears." *Teaching Children Mathematics*.

Blanton, M., & Kaput, J. L. (2002). Characterizing a classroom practice that promotes algebraic reasoning. Working manuscript. Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science. University of Wisconsin–Madison.

Building a foundation for learning algebra in the elementary grades. (2000, Fall). *in Brief*. (Available at http://www.wcer.wisc.edu/ncisla/publications)

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32 (1), 9-13*.

Kaput, J. (1999). Teaching and learning a new algebra. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding (pp. 133-155)*. Mahwah, NJ: Erlbaum.

Swetz, F. J. (1987). *Capitalism and arithmetic: The new math of the 15th century, including the full text of the Treviso arithmetic of 1478* (Trans. by D. E. Smith). La Salle, IL: Open Court.

Yaremia, C. H., Adams, R. H., & Cagle R. (2000). A teacher's 'try' angles. *Teaching Children Mathematics, 6 (5), 299-302*.

NATIONAL CENTER FOR IMPROVING STUDENT LEARNING & ACHIEVEMENT IN MATHEMATICS & SCIENCE
Wisconsin Center for Education Research
School of Education, University of Wisconsin–Madison
1025 West Johnson Street, Madison, WI 53706

Nonprofit Organization
U.S. POSTAGE
PAID
Madison, Wisconsin
Permit No. 658

NIQUI BECKRUM
DATABASE COORD
ERIC/CSMEE
1929 KENNY RD
COLUMBUS OH 43210-1015

BEST COPY AVAILABLE

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

NOTICE

Reproduction Basis

[ ] This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

[X] This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").